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Diff. Calculus (Curvature)

Q.1. For the curve $s = \sqrt{8ay}$, prove that

$$\rho = 4a\sqrt{1 - \frac{y}{2a}}$$

Soln: The equation of the curve is $s = \sqrt{8ay}$ —— (I)

Differentiating with respect to y, we get

$$\frac{ds}{dy} = \sqrt{8a} \times \frac{1}{2\sqrt{y}} = \sqrt{\frac{2a}{y}} = \frac{\sqrt{2a}}{s} \times \sqrt{8a} = \frac{4a}{s} \quad [\text{from I}]$$

But $\frac{ds}{dy} = \operatorname{cosec}\psi$. $\therefore \operatorname{cosec}\psi = \frac{4a}{s}$

Differentiating with respect to s, we get

$$-\operatorname{cosec}\psi \cdot \cot\psi \frac{d\psi}{ds} = -\frac{4a}{s^2}$$

But $\rho = \frac{ds}{d\psi}$

$$\therefore \operatorname{cosec}\psi \sqrt{\operatorname{cosec}^2\psi - 1} \; \frac{1}{\rho} = \frac{4a}{s^2}$$

Hence, $\rho = \frac{s^2}{4a} \times \frac{4a}{s} \sqrt{\frac{16a^2}{s^2} - 1}$

$$= \sqrt{16a^2 - s^2} = 4a\sqrt{1 - \frac{s^2}{16a^2}}$$

$$= 4a\sqrt{1 - \frac{8ay}{16a^2}} \quad [\text{from I}]$$

$$\rho = 4a\sqrt{1 - \frac{y}{2a}} \quad [\because \rho \text{ be the radius of curvature}]$$

Q.2. Show that, for the curve $s = ae^{\frac{x}{c}}$, $c\rho = s\sqrt{(s^2 - c^2)}$, where ρ stands for the radius of curvature at any point of the curve.

Soln: We have, $s = ae^{\frac{x}{c}}$ —— (I)

Differentiating (I) with respect to x, we get

$$\frac{ds}{dx} = \frac{a}{c} e^{\frac{x}{c}} = \frac{s}{c} \quad [\text{by I}]$$

But $\frac{ds}{dx} = \sec\psi \quad \therefore \sec\psi = \frac{s}{c}$ ——— (II)

Differentiating II with respect to s, we get

$$\sec\psi . \tan\psi \frac{d\psi}{ds} = \frac{1}{c} \text{ ——— III}$$

But $\rho = \frac{ds}{d\psi}$.

$$\therefore \sec\psi \sqrt{\sec^2\psi - 1}\, \frac{1}{\rho} = \frac{1}{c} \quad [\text{from III}]$$

Hence, $\rho = s\sqrt{\frac{s^2}{c^2} - 1}$, [from II]

$$\Rightarrow c\rho = s\sqrt{s^2 - c^2}$$ Proved

Q.3. For the curve, $y^2 = s^2 + c^2$, prove that
$s = c\tan\psi$ and $\rho = \frac{y^2}{c}$

Soln: We have, $y^2 = s^2 + c^2$ ——— (I)

Differentiating w.r. to x, we get

$$2y\frac{dy}{dx} = 2s\frac{ds}{dx} \quad \therefore y\tan\psi = s\sec\psi$$

i.e. $y\frac{\sin\psi}{\cos\psi} = \frac{s}{\cos\psi} \quad \therefore \sin\psi = \frac{s}{y}$ ——— (II)

Now, $\tan\psi = \frac{\sin\psi}{\cos\psi} = \frac{\sin\psi}{\sqrt{1-\sin^2\psi}} = \frac{\frac{s}{y}}{\sqrt{1-\frac{s^2}{y^2}}} = \frac{s}{\sqrt{y^2-s^2}} = \frac{s}{\sqrt{c^2}} = \frac{s}{c}$ [by I]

Hence, $s = c\tan\psi$ ——— III

Diff. w.r. to ψ, we get

$$\frac{ds}{d\psi} = c\sec^2\psi = c\left(\sqrt{1+\tan^2\psi}\right)^2 = c(1+\tan^2\psi) = c\left(1+\frac{s^2}{c^2}\right) \quad [\text{from III}]$$

$$= c\left(\frac{c^2+s^2}{c^2}\right) = \frac{y^2}{c} \quad [\text{from I}]$$

Hence, $\rho = \frac{ds}{d\psi} = \frac{y^2}{c}$ Proved.